On the flavor structure of the littlest Higgs model

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ABSTRACT: We investigate the Yukawa sector for up-like quarks in the Lee's version of the Littlest Higgs model. We derive general quark mass and mixing formulae and study leading order contributions due to non-zero light quark masses. Relying on the unitarity of the generalized quark mixing matrix we obtain corrections to the CKM matrix elements. In this model FCNCs appear at the tree level and using leading order contributions we obtain the FCNC couplings for the up-like quark transitions. In light of recent experimental results on the $D^0 - \bar{D}^0$ transition we make predictions for x_D as well as the $D \to \mu^+ \mu^-$ decay rate. Finally, we discuss probabilities for the $t \to c(u)Z$ transitions relevant for the LHC studies.

Contents

1. Introduction

The existence of the hierarchy problem within the SM stimulated constructions of many models of new physics. In the last three decades supersymmetric models offered appealing solutions to the hierarchy problem, although the existence of susyparticles has not been confirmed experimentally. During the last few years, the Little Higgs models [1 – 5] have attracted a lot of attention offering an alternative solution to the hierarchy problem. The main features of all Little Higgs-like models are that Higgs fields appear as Goldstone bosons of a global symmetry broken at some new scale. Then they acquire masses and become pseudo-Goldstone bosons via symmetry breaking at the electroweak scale. The quadratic divergences in the Higgs mass due to the SM gauge bosons are canceled by the contributions of the new heavy gauge bosons with spin 1. The divergence due to the top quark is canceled by the contribution of the new heavy vector-like quark with the charge 2/3 and spin 1/2.

In the simplest model (named the Littlest Higgs model) which has been studied extensively in the literature [6 – 8] the masses of u, d, s, c and b quarks are usually neglected in comparison with the electroweak symmetry breaking scale. Consequently, some tree-level FCNCs appear in the up-quark sector, but only coupling the new heavy quark to the top quark and the Z boson. At the same time only V_{tb} CKM matrix element receives small corrections due to CKM non-unitarity. In a generalization of that model given by Lee [9] mixing of the lighter quarks with the top quark is present. There are two interesting consequences that appear in such a scenario. It allows for Z-mediated FCNCs at the tree-level in the whole up-quark sector (while not in the down-quark sector). It also extends the 3×3 CKM matrix in the SM to a 4×3 matrix and introduces non-unitarity corrections to all of the CKM matrix elements. Recently, Chen et al. [10] have discussed $D - \bar{D}$ mixing

in a similar model, but only after imposing additional assumptions. Namely, in order to preserve the large up-quark mass hierarchy they assume a special form of the Yukawa matrices, allowing them to constrain the model parameters. Using present errors in the CKM matrix elements they are able to induce rather large flavor changing effects.

Motivated by the results of these papers we re-investigate the flavor structure of the Littlest Higgs model (LHM). We perform an eigensystem analysis of the more general LHM up-quark mass matrix and are able to recover the results of the constrained model [6] as well as give robust predictions for the more general case. After Introduction, we give a general analysis of the up-quark Yukawa couplings in section II. Section III contains analysis of CKM unitarity and FCNCs. Phenomenological consequences are discussed in section IV, while conclusions are given in section V.

2. LHM up Yukawas and CP violation

We first focus on the simplest LHM, whose phenomenology was first studied by Han et al. [6]. The light and heavy top quark Yukawa sector of this model is given by eq. (24) of [6]:

$$\mathcal{L}_Y = \frac{1}{2}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\chi_i \Sigma_{jx}\Sigma_{ky}u_3^{'c} + \lambda_2 f\tilde{t}\tilde{t}^{'c} + \text{H.c.}, \tag{2.1}$$

where $\chi^T = (b_3, t_3, \tilde{t})$, ϵ_{ijk} and ϵ_{xy} are antisymmetric tensors, with $ijk = 1, 2, 3$ and $xy = 4, 5$. Σ contains the Higgs fields in the adjoint representation of the global LH SU(5) (c.f. [6] eq. (3)), $u_3^{'c}$ and $\tilde{t}^{'c}$ are the two right-handed top fields, while f is the VEV of the heavy Higgs ($f \simeq 1$ TeV). Note that $\lambda_{1,2}$ are $c-$numbers in this model implying absence of mixing of the third generation with the first two generations in the up sector. However, in this form, the model is also CP conserving, as can be easily deduced by studying weak basis invariants of the resulting mass matrix [12, 11]. CP is preserved even in presence of non-diagonal Yukawa terms involving only the first two generations of up-quarks and regardless of the down-quark sector. In order to provide SM-like sources of CP violation, one must therefore add further non-diagonal Yukawa terms, mixing the light top quark with the first two generations, but necessarily not involving the heavy top quark. If we require that the one-loop top quark contributions to the Higgs mass largely vanish, these additional Yukawa couplings (we denote them by λ_{ij}^u) must be much smaller than λ_1. This leads to a generalized up-quark mass matrix in the weak basis

$$\mathcal{M}_p = \begin{pmatrix} iv\lambda_{11}^u & iv\lambda_{12}^u & iv\lambda_{13}^u & 0 \\ iv\lambda_{21}^u & iv\lambda_{22}^u & iv\lambda_{23}^u & 0 \\ iv\lambda_{31}^u & iv\lambda_{32}^u & iv(\lambda_1 + \lambda_{33}^u) & 0 \\ 0 & 0 & f\lambda_1 & f\lambda_2 \end{pmatrix}. \tag{2.2}$$

Lee [9] similarly generalizes the Yukawa part of the model by including a general mixing pattern in the up-quark sector. In eq. (2.15) of [9] he writes

$$\mathcal{L}_Y = \frac{1}{2}\lambda_1^{ab} f \epsilon_{ijk}\epsilon_{xy}\chi_{ai} \Sigma_{jx}\Sigma_{ky}u_b^{'c} + \lambda_2 f\tilde{t}\tilde{t}^{'c} + \text{H.c.}, \tag{2.3}$$

with $ab = 1, 2, 3$ and $\chi_i^T = (b_i, t_i, \delta_{i3}\tilde{t})$. Then the up-quark mass matrix in the weak basis should become[1]

$$\mathcal{M}_p = \begin{pmatrix} iv\lambda_1^{11} & iv\lambda_1^{12} & iv\lambda_1^{13} & 0 \\ iv\lambda_1^{21} & iv\lambda_1^{22} & iv\lambda_1^{23} & 0 \\ iv\lambda_1^{31} & iv\lambda_1^{32} & iv\lambda_1^{33} & 0 \\ f\lambda_1^{31} & f\lambda_1^{32} & f\lambda_1^{33} & f\lambda_2 \end{pmatrix}. \tag{2.4}$$

We see that the mass matrices in the two models apparently differ in the form of their fourth rows. However, when requiring (partial) cancelation of top quark contributions to the Higgs mass, both models can be treated equivalently.

We perform an eigensystem analysis of this quark mass sector based on the conjugated versions of eqs. (24.26) of ref. [11]. Namely we can denote

$$\mathcal{M}_p^\dagger = \begin{pmatrix} G_{p(3\times3)} & J_{p(3\times1)} \\ 0 & \hat{M}_p \end{pmatrix}, \tag{2.5}$$

while the down-quark mass matrix M_n is general three-by-three and complex. $\hat{M}_p$ is a $c-$value and can be made real via suitable phase redefinition of the heavy top field $\tilde{t}'^c$, while G_p can be made diagonal and real via suitable weak basis transformations $(G_p \to \mathrm{diag}(v\eta_1, v\eta_2, v\eta_3))$. The unitary transformations involved induce corrections to J_p in terms of mixing of components which we since denote with tilde: $J_p^T = (f\tilde{\lambda}_1^{31}, f\tilde{\lambda}_1^{32}, f\tilde{\lambda}_1^{33})$. $\tilde{\lambda}_1^{3i} = \sum_j L_{ij}\lambda_1^{(3j)}$ with L_{ij} being components of a unitary matrix diagonalizing G_p so that $\sum_j |L_{ij}|^2 = 1$ for any i. We see that the end form of $\mathcal{M}_p$ is qualitatively the same for both models under consideration. The mass eigenvalue equation $\mathcal{M}_p\mathcal{M}_p^\dagger W_p = W_p D_p^2$, where

$$W_p = \begin{pmatrix} K_{p(3\times3)} & R_{p(3\times1)} \\ S_{p(1\times3)} & T_p \end{pmatrix} \tag{2.6}$$

is a unitary eigenvector matrix and $D_p = \mathrm{diag}[\bar{m}_{p(3\times3)}, \bar{M}_p]$ is the diagonal eigenmass matrix $(\bar{m}_p = \mathrm{diag}(m_1, m_2, m_3))$, can then be written as a set of matrix equations [11]

$$G_p^\dagger G_p K_p + G_p^\dagger J_p S_p = K_p \bar{m}_p^2, \tag{2.7a}$$

$$G_p^\dagger G_p R_p + G_p^\dagger J_p T_p = R_p \bar{M}_p^2, \tag{2.7b}$$

$$J_p^\dagger G_p K_p + (J_p^\dagger J_p + M_p^2)S_p = S_p \bar{m}_p^2, \tag{2.7c}$$

$$J_p^\dagger G_p R_p + (J_p^\dagger J_p + M_p^2)T_p = T_p \bar{M}_p^2, \tag{2.7d}$$

while the W_p unitary constraint relevant for this discussion reads

$$R_p^\dagger R_p + T_p^* T_p = 1. \tag{2.8}$$

We start by evaluating eqs. (2.7b) and (2.7d):

$$R_p \bar{M}_p^2 = v^2 \mathrm{diag}(\eta_1^2, \eta_2^2, \eta_3^2)R_p$$
$$+ vf(\eta_1\tilde{\lambda}_1^{31}, \eta_2\tilde{\lambda}_1^{32}, \eta_3\tilde{\lambda}_1^{33})^T T_p, \tag{2.9a}$$

$$T_p \bar{M}_p^2 = vf(\eta_1\tilde{\lambda}_1^{31*}, \eta_2\tilde{\lambda}_1^{32*}, \eta_3\tilde{\lambda}_1^{33*})R_p$$
$$+ f^2|\lambda|^2 T_p, \tag{2.9b}$$

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[1]We find a difference in the fourth row calculation of ref. [9].

where $|\lambda|^2 = (|\tilde{\lambda}_1^{31}|^2 + |\tilde{\lambda}_1^{32}|^2 + |\tilde{\lambda}_1^{33}|^2 + |\lambda_2|^2)$. We notice that requiring the heavy top mass to scale as $\bar{M}_p \sim f$ the two equations can be solved simultaneously provided $R_p \lesssim T_p$ in terms of v/f scaling. Then, to leading order in v/f, the heavy top mass is

$$\bar{M}_p^2 = |\lambda|^2 f^2, \tag{2.10}$$

while for R_p and T_p we get

$$R_p = \frac{v}{f}\frac{1}{|\lambda|^2}(\eta_1\tilde{\lambda}_1^{31}, \eta_2\tilde{\lambda}_1^{32}, \eta_3\tilde{\lambda}_1^{33})^T T_p, \tag{2.11a}$$

$$|T_p| \simeq 1 - \frac{1}{2}R_p^\dagger R_p = 1 - \mathcal{O}(v/f)^2, \tag{2.11b}$$

where the unitarity constraint together with v/f expansion of the square root has been used in the last line.

Next we evaluate eqs. (2.7a and 2.7c)

$$K_p\bar{m}_p^2 = v^2\mathrm{diag}(\eta_1^2, \eta_2^2, \eta_3^2)K_p + vf(\eta_1\tilde{\lambda}_1^{31}, \eta_2\tilde{\lambda}_1^{32}, \eta_3\tilde{\lambda}_1^{33})^T S_p, \tag{2.12a}$$

$$S_p\bar{m}_p^2 = vf(\eta_1\tilde{\lambda}_1^{31*}, \eta_2\tilde{\lambda}_1^{32*}, \eta_3\tilde{\lambda}_1^{33*})K_p + f^2|\lambda|^2 S_p. \tag{2.12b}$$

Requiring the light up-quark mass eigenvalues to scale as $\bar{m}_p \sim v$, we can solve both equations without any fine-tuning provided S_p and K_p have fixed relative scaling in v/f: $S_p \sim K_p v/f$. Then the left hand side of eq. (2.12b) is of higher order in v/f than the right hand side and can be neglected yielding the relation

$$S_p = -\frac{v}{f}\frac{1}{|\lambda|^2}(\eta_1\tilde{\lambda}_1^{31*}, \eta_2\tilde{\lambda}_1^{32*}, \eta_3\tilde{\lambda}_1^{33*})K_p. \tag{2.13}$$

Inserting this expression into eq. (2.12a) yields[2]

$$K_p\mathrm{diag}(m_1^2, m_2^2, m_3^2) = v^2\left[\mathrm{diag}(\eta_1^2, \eta_2^2, \eta_3^2) - \left(\frac{\eta_i\eta_j\tilde{\lambda}_1^{3i}\tilde{\lambda}_1^{3j*}}{|\lambda|^2}\right)_{(3\times3)}\right]K_p, \tag{2.14}$$

where in this short-hand matrix notation there is *no* summation over the repeated quark generation indices. The above matrix equation in full form is given in the appendix. Next we notice that the off-diagonal elements of the matrix multiplying K_p on the right hand side of eq. (2.14) are generally smaller than the diagonal ones and tend to zero with $\tilde{\lambda}_1^{i3}/\lambda_2 \to 0$. Therefore we approximate the solution, unitary at leading order in v/f, with a linear expansion around the diagonal, yielding

$$m_i^2 = v^2\eta_i^2\left[1 - \frac{|\tilde{\lambda}_1^{3i}|^2}{|\lambda|^2}\right], \tag{2.15a}$$

$$(K_p)_{ij} = \delta_{ij} + (\delta_{ij} - 1)\frac{v^2\eta_i\eta_j\tilde{\lambda}_1^{3i}\tilde{\lambda}_1^{3j*}}{(m_i^2 - m_j^2)|\lambda|^2}. \tag{2.15b}$$

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[2]The appearance of the off-diagonal contributions in eq. (2.14) is the direct consequence and main difference due the different v/f scaling of J_p with respect to the one in section 24.3 of ref. [11].

Again in eq. (2.15b) there is no summation over repeated quark generation indices and the full matrix form of K_p in this approximation is given in the appendix. With $\tilde{\lambda}_1^{31} = \tilde{\lambda}_1^{32} = 0$ and $\tilde{\lambda}_1^{33} = \eta_3 = \lambda_1$ we reproduce the usual result for the light and heavy top masses in the simplest model of Han et al. [6] which ensures exact cancelation of top-quark contributions to the Higgs mass at one loop. Deviations from this limit in terms of non-vanishing $\tilde{\lambda}_1^{31}$ and $\tilde{\lambda}_1^{32}$ on one side reintroduce such corrections, while on the other side they provide needed sources of SM-like CP violation.

3. CKM unitarity and FCNCs

FCNCs at tree level via flavor changing Z couplings can be easily deduced by evaluating $Z_p = A_p^\dagger A_p$, where A_p are the first three rows of W_p or $A_p = (K_p, R_p)$. Then the FCNC of up-like quarks coupling to the Z boson is $J_\mu^{FC} = (g/2c_W)\bar{u}_{Li}\gamma_\mu(Z_p)_{ij}u_{Lj}$, where g is the $SU(2)_L$ gauge coupling and c_W is the cosine of the Weinberg angle. At leading order in v/f we get off-diagonal elements of Z_p only in the fourth column (and row)

$$(Z_p)_{i4} = \sum_j (K_p)_{ji}^*(R_p)_j \,. \tag{3.1}$$

FCNCs among the light up-type quarks only come at the order of $(v/f)^2$, are due to 4×4 up-quark basis unitarity [11] and yield

$$(Z_p)_{ij} = \delta_{ij} - (Z_p)_{i4}^*(Z_p)_{j4}. \tag{3.2}$$

At the same time we get CKM non-unitary corrections in terms of fourth row CKM matrix elements, which can be calculated via $V_{CKM} = A_p^\dagger A_n$, where A_n is the 3×3 down quark unitary mixing matrix. In absence of fourth row entries in $\mathcal{M}_p$ due to the mixing with the vector top quark, A_p would just be the identity and the usual form of $V_{CKM} = A_n$ would be obtained. Now however, we obtain for the fourth row CKM matrix elements

$$(V_{CKM})_{4i} = \sum_k (R_p)_k^*(A_n)_{ki} \,, \tag{3.3}$$

while the 3×3 non-unitary mixing sub-matrix for the light quarks is, again due to 4×4 unitarity

$$(V_{CKM})_{ij} = \sum_k (K_p)_{ki}^*(A_n)_{kj} - (V_{CKM})_{4i}^*(V_{CKM})_{4j} \,. \tag{3.4}$$

Formulae (3.1)–(3.4) are exact up to v/f corrections, but more importantly *regardless of any approximations to the solution for K_p from eq. (2.14)*, thus representing faithfully the generally rich flavor structure of the LH model.

Our treatment leads to qualitatively similar conclusions as found in [9] regarding FCNCs, but we disagree in the procedure as well as in the form of the final results. The approach of [10] on the other hand imposes fine-tuning cancelations among up-quark Yukawa elements (i.e. requiring cancelation of the two terms in the square brackets in (2.15a) for the first two generations) in order to obtain relations among them. However not all parameters feature independently in the mass formulae. By identifying the heavy top mass

$m_T = f\sqrt{|\lambda^2|}$, we find that all expressions only depend on certain combinations: $(v\eta_i)$ and $e_i \equiv \tilde{\lambda}_1^{3i}/\sqrt{|\lambda^2|}$. Using the first, we can absorb all light Higgs VEV dependence into light quark masses and mixings, while the second indicates that phenomenologically, the LH FCNC couplings lie on three-plane intersection of a four-sphere with radius $\sqrt{|\lambda^2|}$. Therefore we parameterize the moduli of e_i using generalized Euler's angles, projected on the three-plane (distance from the origin is parameterized by $\sin\gamma$) α, β, γ: $|e_1| = |s_\alpha s_\beta s_\gamma|$, $|e_2| = |c_\alpha s_\beta s_\gamma|$, $|e_3| = |c_\beta s_\gamma|$, where $s_x = \sin x$ and $c_x = \cos x$. Note that, although $|e_i|$ are bounded to lie between 0 and 1, providing sources of SM like CP violation discussed in the previous section requires at least two of them to be different from zero (the constrained model of Han corresponds to $c_\beta = 1$ or $e_1 = e_2 = 0$). At the same time, due to the orthogonality of projections c_x and s_x, only one of the $|e_i|$ can be set close to 1 at best, while in addition cancelation of top loop contributions to the Higgs mass requires $|e_3|$ to be much larger than $|e_{1,2}|$. This eventually rules out a simultaneous mass cancelation via fine-tuning for the first two generations in eq. (2.15a).

More explicit analytic expressions for CKM corrections and FCNCs in closed form can then be obtained by keeping only the leading order terms in the off-diagonal expansion of K_p (i.e. using solutions (2.15a) and (2.15b)) in which case our analysis reverts to the one of ref. [11]. We obtain

$$(Z_p)_{i4} \simeq \frac{m_i}{m_T} \frac{e_i}{\sqrt{1-|e_i|^2}} \simeq (R_p)_i \,, \tag{3.5a}$$

$$(Z_p)_{ij} \simeq \delta_{ij} - \frac{m_i m_j}{m_T^2} \frac{e_i^*}{\sqrt{1-|e_i|^2}} \frac{e_j}{\sqrt{1-|e_j|^2}} \,, \tag{3.5b}$$

$$(V_{CKM})_{4i} \simeq \sum_k \frac{m_k}{m_T} (A_n)_{ki} \frac{e_k^*}{\sqrt{1-|e_k|^2}}$$

$$\simeq \sum_k \frac{m_k}{m_T} (V_{CKM})_{ki} \frac{e_k^*}{\sqrt{1-|e_k|^2}} \,, \tag{3.5c}$$

$$(V_{CKM})_{ij} \simeq (A_n)_{ij} - (V_{CKM})_{4i}^* (V_{CKM})_{4j} \,. \tag{3.5d}$$

Actually, due to the large hierarchy in the up quark masses, expansion (2.15b) is always a good approximation for K_p. This can be seen by parameterizing the off-diagonal elements of K_p in eq. (2.15b) or (A.3) in terms of generalized Euler's angles and physical quark masses. Then due to the orthogonality of the projections c_i, s_i expressions of the type $e_i^* e_j / \sqrt{1-|e_i|^2}\sqrt{1-|e_j|^2}$ for $i \neq j$ are always bounded from above by 1, while off-diagonal elements in K_p are in addition suppressed by small ratios of quark masses among different generations. Therefore, even if the eigenvalues in eq. (2.15a) receive relatively large corrections, these are not reflected in large deviations from the diagonality in K_p and consequently in FCNCs as we will see in the next section.

4. Phenomenology

We first calculate FCNC constraints, given experimentally from CKM non-unitarity [9]. We take the current bounds on the CKM moduli, obtained from tree level processes without referring to 3×3 CKM unitarity [13]. Then the complete 4×4 mixing matrix unitarity
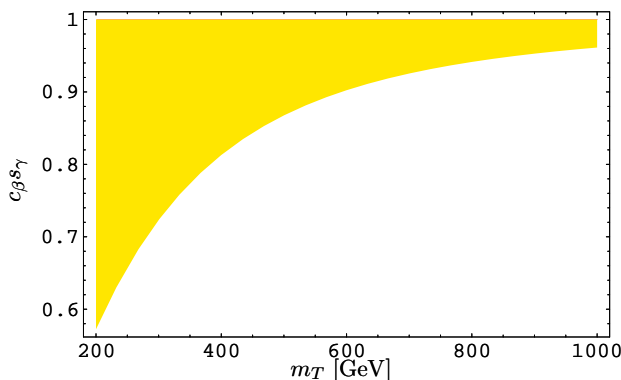
Figure 1: LHM parameter plane spanned by m_T and $c_\beta s_\gamma$. The shaded region in yellow(grey) is excluded by present CKM unitarity bounds as explained in the text.

conditions constrain FCNCs through the relation $Z_p = V_{CKM}V_{CKM}^\dagger$ [11]. We notice that the stringiest unitarity bounds on the parameters will come from the top sector due to large up-quark mass hierarchy, and from the diagonal elements, where the couplings are not bounded by orthogonality conditions. In particular, the most constraining is the recent direct lower bound on the magnitude of the V_{tb} CKM matrix element from the D0 collaboration [14] when interpreted in the general models with vector-like singlet top quarks [15] yielding $|V_{tb}| > 0.71$. We write down the most perspective constraints

$$|(Z_p)_{33}| = \left|1 - \frac{m_t^2}{m_T^2}\frac{c_\beta^2 s_\gamma^2}{(1 - c_\beta^2 s_\gamma^2)}\right| > 0.52\,, \tag{4.1a}$$

$$|(Z_p)_{32}| = \left|\frac{m_t m_c}{m_T^2}\frac{s_\alpha c_\beta s_\beta s_\gamma^2}{\sqrt{1 - s_\alpha^2 s_\beta^2 s_\gamma^2}\sqrt{1 - c_\beta^2 s_\gamma^2}}\right| < 0.13\,, \tag{4.1b}$$

$$|(Z_p)_{31}| = \left|\frac{m_t m_u}{m_T^2}\frac{c_\alpha c_\beta s_\beta s_\gamma^2}{\sqrt{1 - c_\alpha^2 s_\beta^2 s_\gamma^2}\sqrt{1 - c_\beta^2 s_\gamma^2}}\right| < 0.11\,, \tag{4.1c}$$

$$|(Z_p)_{22}| = \left|1 - \frac{m_c^2}{m_T^2}\frac{s_\alpha^2 s_\beta^2 s_\gamma^2}{1 - s_\alpha^2 s_\beta^2 s_\gamma^2}\right| > 0.63\,, \tag{4.1d}$$

$$|(Z_p)_{11}| = \left|1 - \frac{m_u^2}{m_T^2}\frac{c_\alpha^2 s_\beta^2 s_\gamma^2}{1 - c_\alpha^2 s_\beta^2 s_\gamma^2}\right| > 0.996\,. \tag{4.1e}$$

Presently only eq. (4.1a) is restrictive enough to be used as any kind of constraint on the parameters of the model. It excludes a region in the parameter plane spanned by m_T and $c_\beta s_\gamma$ (corresponding to $\lambda_1/\sqrt{\lambda_1^2 + \lambda_2^2}$ in the constrained model) as shown on figure 1.

We see that for heavy top-quark masses above $1\,\mathrm{TeV}$, even this bound is ineffective at present.

Next we study $D - \bar{D}$ mixing. A general study of new physics contributions in this process was recently performed in [16]. For $x_D = \Delta m_D/\Gamma_D$ contribution due to Z mediated

FCNCs we use the known form

$$x_D = \frac{\sqrt{2}m_D}{3\Gamma_D} G_F f_D^2 B_D |(Z_p)_{12}|^2 r_1(m_c, m_Z), \tag{4.2}$$

where the function $r_1(\mu, M) = [\alpha_s(M)/\alpha_s(m_b)]^{6/23} \times [\alpha_s(m_b)/\alpha_s(\mu)]^{6/25}$ accounts for the one-loop QCD running, G_F is the Fermi constant, f_D is the D meson decay constant and B_D is the D meson bag parameter. In our numerical evaluation we use PDG [13] values for quark and Z boson masses, mass and width of the D meson, G_F and $\alpha_s(m_Z)$, while for the hadronic parameters we take $f_D = 0.22$ GeV [17] from CLEO-c measurement and $B_D = 0.82$ [18] from a quenched lattice study. After evaluating these known quantities we obtain

$$\begin{aligned} x_D &= 2 \times 10^5 |(Z_p)_{12}|^2 \\ &\simeq 3 \times 10^{-12} \left| \frac{s_\alpha c_\alpha s_\beta^2 s_\gamma^2}{\sqrt{1 - c_\alpha^2 s_\beta^2 s_\gamma^2}\sqrt{1 - s_\alpha^2 s_\beta^2 s_\gamma^2}} \left(\frac{1 \text{ TeV}}{m_T}\right)^2 \right|^2. \end{aligned} \tag{4.3}$$

We have to compare this expression with the recent experimental results from the B-factories [19, 20], which give a value of $x_D = 0.0087 \pm 0.003$ [21]. Similarly for the rare $D \to \mu^+\mu^-$ decay width, we use the known form for Z mediated FCNC contribution

$$\Gamma(D^0 \to \mu^+\mu^-) = \frac{m_D}{64\pi} \left(\frac{G_F}{\sqrt{2}}\right)^2 |(Z_p)_{12}|^2 f_D^2 m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_D^2}}, \tag{4.4}$$

and obtain

$$\begin{aligned} \mathcal{B}r(D^0 \to \mu^+\mu^-) &= 3 \times 10^{-4} |(Z_p)_{12}|^2 \\ &\simeq 3 \times 10^{-21} \left| \frac{s_\alpha c_\alpha s_\beta^2 s_\gamma^2}{\sqrt{1 - c_\alpha^2 s_\beta^2 s_\gamma^2}\sqrt{1 - s_\alpha^2 s_\beta^2 s_\gamma^2}} \left(\frac{1 \text{ TeV}}{m_T}\right)^2 \right|^2, \end{aligned} \tag{4.5}$$

again to be compared to the current experimental limit $\mathcal{B}r(D^0 \to \ell^+\ell^-) < 1.2 \times 10^{-6}$ [22] from BaBar. We see that in both processes, the LHM contributions at tree level are negligible. Note however, that due to the same FC Z coupling appearing in both eqs. (4.3) and (4.5) a general upper bound prediction for the $Br(D^0 \to \mu^+\mu^-)$ mediated by such effective couplings can be made. Namely, saturating the measured value of x_D with the short distance contribution in the first line of eq. (4.3) we obtain an upper bound on $|(Z_p)_{12}| < 2 \times 10^{-4}$ and consequently $Br(D^0 \to \mu^+\mu^-)_{Z_p} < 2 \times 10^{-11}$. The rare D decays due to $c \to uZ$ transitions are then also very suppressed as already noticed in [23, 21]. Therefore we only give predictions for the $t \to cZ$ and $t \to uZ$ decay rates. In the SM these transitions are highly suppressed and their branching ratios are of the order $\mathcal{O}(10^{-10})$ or less [24]. On the other hand, current experimental constraints on these transitions are not very strong [25]. Following [24, 9], we normalize the decay width

$$\Gamma(t \to c(u)Z) = \frac{m_t^3}{16\pi} \frac{G_F}{\sqrt{2}} |(Z_p)_{32(1)}|^2 f(x_Z, x_c), \tag{4.6}$$

where $f(x, y) = [(1 - y)^2 - 2x^2 + x(1 + y)]\lambda^{1/2}(x, y)$, $\lambda^{1/2}(x, y) = \sqrt{1 + y^2 + x^2 - 2xy - 2x - 2y}$
and $x_i = m_i^2/m_t^2$, to the dominant $t \to bW$ decay rate [26, 24]

$$\Gamma(t \to bW) = \frac{m_t^3}{8\pi} \frac{G_F}{\sqrt{2}} |V_{tb}|^2 f(x_W, x_b), \tag{4.7}$$

and obtain for the branching ratios approximately

$$\mathcal{B}r(t \to cZ) \lesssim 0.5 \left| \frac{(Z_p)_{32}}{V_{tb}} \right|^2$$

$$\simeq 5 \times 10^{-8} \left| \frac{s_\alpha c_\beta s_\beta s_\gamma^2}{\sqrt{1 - c_\beta^2 s_\gamma^2} \sqrt{1 - s_\alpha^2 s_\beta^2 s_\gamma^2}} \left(\frac{1 \text{ TeV}}{m_T} \right)^2 \right|^2, \tag{4.8}$$

and

$$\mathcal{B}r(t \to uZ) \lesssim 0.5 \left| \frac{(Z_p)_{31}}{V_{tb}} \right|^2$$

$$\simeq 2 \times 10^{-13} \left| \frac{c_\alpha c_\beta s_\beta s_\gamma^2}{\sqrt{1 - c_\alpha^2 s_\beta^2 s_\gamma^2} \sqrt{1 - c_\beta^2 s_\gamma^2}} \left(\frac{1 \text{ TeV}}{m_T} \right)^2 \right|^2, \tag{4.9}$$

where in the last lines of eqs. (4.8) and (4.9) we have again used the lower bound on $|V_{tb}|$ from [15].

5. Conclusions

We have reinvestigated the LH model of Lee [9] by applying general constraints on extra vector-like quark singlet models given in ref. [11]. Namely, we have discussed the appearance of tree level FCNCs and CKM unitarity violation in a LHM with general Yukawa couplings and shown that, contrary to previous conclusions, the up-quark flavor changing Z couplings are *not* proportional to the CKM matrix elements. Instead they are proportional to ratios of up-quark masses relative to the heavy top quark mass and can be parameterized in terms of three new angle parameters. Due to the large constraints on the heavy top quark mass, these tree level contributions are found to be negligible even when compared to SM loop contributions. Contrary to the derivation of Chen et al. [10], we do not impose any fine-tuning and cancelations among the various Yukawa matrix elements in order to obtain the measured up-quark masses. On the other hand, our analysis shows, that mass relation between the light and heavy top quark, ensuring the exact cancelation of one-loop contributions to the Higgs mass, is not maintained in the general model. Relaxing this requirement could have important effects on the currently established heavy top quark mass limits from low energy phenomenology.

Acknowledgments

A. Full forms of quark mass diagonalization formulae

Here we give the matrix formulae given in short-hand notation in eqs. (2.14) and (2.15b) in their full form and by using the parametrization in terms of $e_i = \tilde{\lambda}_1^{3i}/|\lambda|$ parameters. Matrix eq. (2.14) for K_p reads

$$
K_p \cdot \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} = v^2 \begin{pmatrix} \eta_1^2(1-|e_1|^2) & -\eta_1\eta_2 e_1 e_2^* & -\eta_1\eta_3 e_1 e_3^* \\ -\eta_1\eta_2 e_2 e_1^* & \eta_2^2(1-|e_2|^2) & -\eta_2\eta_3 e_2 e_3^* \\ -\eta_1\eta_3 e_3 e_1^* & -\eta_2\eta_3 e_3 e_2^* & \eta_3^2(1-|e_3|^2) \end{pmatrix} \cdot K_p, \quad \text{(A.1)}
$$

with the approximate solutions for K_p of the form

$$
K_p = \begin{pmatrix} 1 & -\dfrac{v^2\eta_1\eta_2 e_1 e_2^*}{(m_1^2-m_2^2)} & -\dfrac{v^2\eta_1\eta_3 e_1 e_3^*}{(m_1^2-m_3^2)} \\ -\dfrac{v^2\eta_1\eta_2 e_2 e_1^*}{(m_2^2-m_1^2)} & 1 & -\dfrac{v^2\eta_2\eta_3 e_2 e_3^*}{(m_2^2-m_3^2)} \\ -\dfrac{v^2\eta_1\eta_2 e_3 e_1^*}{(m_3^2-m_1^2)} & -\dfrac{v^2\eta_2\eta_3 e_3 e_2^*}{(m_3^2-m_2^2)} & 1 \end{pmatrix}. \quad \text{(A.2)}
$$

We remaind the reader that in this approximation the light up-quark masses are given by $m_i = v\eta_i\sqrt{1-|e_i|^2}$. Then due to the large measured mass hierarchy in the up-quark sector we have approximately

$$
K_p \simeq \begin{pmatrix} 1 & \dfrac{m_1}{m_2}\hat{e}_1\hat{e}_2^* & \dfrac{m_1}{m_3}\hat{e}_1\hat{e}_3^* \\ -\dfrac{m_1}{m_2}\hat{e}_2\hat{e}_1^* & 1 & \dfrac{m_2}{m_3}\hat{e}_2\hat{e}_3^* \\ -\dfrac{m_1}{m_3}\hat{e}_3\hat{e}_1^* & -\dfrac{m_2}{m_3}\hat{e}_3\hat{e}_2^* & 1 \end{pmatrix}, \quad \text{(A.3)}
$$

where we have used $\hat{e}_i = e_i/\sqrt{1-|e_i|^2}$.

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